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EQUATIONS OF CONVECTIVE INSTABILITY OF A TWO-PHASE MEDIUM WITH MODULATION OF THE GRAVITY FORCE

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Thermal convection in a heterogeneous medium consisting of a fluid and solid particles is studied under conditions of finite-frequency vibrations. Equations of convection are derived within the framework of the generalized Boussinesq approximation, and the problem of stability of a horizontal layer to infinitesimal perturbations under the condition of vertical vibrations is considered.

Key words: *particles, vibration, convection, stability.*

Introduction. The study of the influence of periodic changes in one parameter of the medium on the emergence of convection was started in [1], where periodic modulation of an equilibrium temperature gradient was considered. The modulation was shown to have different effects on stability of a nonuniformly heated fluid. At small modulation amplitudes η, vibrations change the critical value of the Rayleigh number R_{cr}, and R_{cr} monotonically increases with increasing modulation amplitude. Vibrations exert a stabilizing effect at small amplitudes only. Beginning from a certain critical value η_*, an increase in amplitude at fixed Rayleigh number and modulation frequency leads to parametric excitation of vibrational convective motions with the period of changes in intensity being multiple to the period of modulation. Parametric destabilization of equilibrium occurs for all values of R (including $\mathrm{R} < 0$ corresponding to heating from above). This fact was validated experimentally in [2, 3].

Convective instability of a single-phase medium was studied in [1–3]. It is known, however, that the flow is usually visualized in experiments with the use of fine particles of aluminum powder, polystyrene, tobacco smoke, etc. Hence, the issue of the influence of particles on the structure and stability of convection has to be considered.

The first attempt to take into account the effect of particles on convection stability was made in [4, 5], where a simple model within the framework of the Boussinesq approximations was proposed. The model was used to analyze the stability of the steady convective flow of a medium containing solid particles between vertical planes heated to different temperatures. Particle sedimentation was neglected in [4], but the problem in [5] was solved in the full statement. As was shown in [6], however, the system of equations derived in [5] is not a rigorously Boussinesq system, because it contains asymptotically large and asymptotically small parameters. A noncontradictory closed system of equations of convection in a medium with solid particles was also obtained in [6] for the first time.

The approach proposed in [6] was used in [7, 8] to derive equations of convection in a fluid with solid particles under conditions of modulation of the gravity field. The problem of flow stability in a vertical layer of the fluid with horizontal vibrations along the layer was also considered there.

At the same time, it may be demonstrated that the equations of convection in a dusty medium derived in [6–8] have substantial restrictions on the magnitude of gradients of the mass concentration of particles. Hence, the problems considered in those papers are well-posed only for a uniform concentration distribution over the volume. Obviously, only qualitative characteristics of the process can be considered in this case, because, for instance, by virtue of the continuity equation, a small-scale modulation of the particle concentration leads to spatial modulation

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of the velocity field where the particles are located. The present study can fill this gap. The problem of stability of a horizontal layer of a two-phase medium under conditions of finite-frequency vertical vibrations is considered with the use of equations derived below.

1. Derivation of Constitutive Equations. Let a layer of a fluid with solid particles perform harmonic vibrations in the direction of the vector $\boldsymbol{n}$ with an amplitude a and frequency ω. In this case, each volume element of the two-phase medium experiences the action of vibrational forces of inertia, in addition to the gravity force. Formally, this means that the transport (vibrational) acceleration is added to the static acceleration due to gravity in a noninertial frame of reference fitted to the layer:

$$\boldsymbol{g} \to \boldsymbol{g} + \boldsymbol{n} a\omega^2 \sin(\omega t).$$

Let μ and φ be the fractions of the fluid and solid phases per, respectively, unit volume of the heterogeneous mixture: $\mu+\varphi = 1$. For each component of the medium, we write the equations of conservation of mass, momentum, and energy in differential form [9]:

$$\frac{\partial(\mu\rho_{\rm f})}{\partial t} + \nabla\cdot(\mu\rho_{\rm f}\boldsymbol{v}_{\rm f}) = 0, \qquad \frac{\partial(\varphi\rho_{\rm s})}{\partial t} + \nabla\cdot(\varphi\rho_{\rm s}\boldsymbol{v}_{\rm s}) = 0,$$

$$\mu\rho_{\rm f}\Big(\frac{\partial \boldsymbol{v}_{\rm f}}{\partial t} + \boldsymbol{v}_{\rm f}\cdot\nabla\boldsymbol{v}_{\rm f}\Big) = -\mu\nabla P + \eta\nabla\cdot(\mu_*\boldsymbol{e}) + \varphi\alpha(\boldsymbol{v}_{\rm s} - \boldsymbol{v}_{\rm f}) - \mu\rho_{\rm f} g(\boldsymbol{\gamma} + \boldsymbol{n}A\sin(\omega t)),$$

$$\varphi\rho_{\rm s}\Big(\frac{\partial \boldsymbol{v}_{\rm s}}{\partial t} + \boldsymbol{v}_{\rm s}\cdot\nabla\boldsymbol{v}_{\rm s}\Big) = -\varphi\nabla P - \varphi\alpha(\boldsymbol{v}_{\rm s} - \boldsymbol{v}_{\rm f}) - \varphi\rho_{\rm s} g(\boldsymbol{\gamma} + \boldsymbol{n}A\sin(\omega t)), \tag{1}$$

$$\mu\rho_{\rm f} C_{\rm f}\Big(\frac{\partial T_{\rm f}}{\partial t} + \boldsymbol{v}_{\rm f}\cdot\nabla T_{\rm f}\Big) = \varkappa\nabla\cdot(\mu\nabla T_{\rm f}) + \varphi\zeta(T_{\rm s} - T_{\rm f}),$$

$$\varphi\rho_{\rm s} C_{\rm s}\Big(\frac{\partial T_{\rm s}}{\partial t} + \boldsymbol{v}_{\rm s}\cdot\nabla T_{\rm s}\Big) = -\varphi\zeta(T_{\rm s} - T_{\rm f}).$$

Here t is the time, P is the pressure, $\boldsymbol{v}_{\rm f}$ and $\boldsymbol{v}_{\rm s}$ are the velocities of the components of the mixture, the subscripts "f" and "s" refer to the fluid and solid phases, respectively, $T_{\rm f}$ and $T_{\rm s}$ are the temperatures of the fluid and solid components, $\rho_{\rm f}$, $\rho_{\rm s}$ and $c_{\rm f}$, $c_{\rm s}$ are their densities and specific heats, respectively, $\varkappa$ is the thermal conductivity of the fluid, μ_* is the viscosity, α and ζ are the coefficients of friction and heat transfer between the phases, $\boldsymbol{\gamma}$ is the unit vector directed along the z axis, $\boldsymbol{e}$ is the viscous stress tensor, and $A = a\omega^2/g$ is the relative amplitude of vibrations. Equations (1) imply that the interaction between the phases follows the Stokes law, and the heat transfer follows the Fourier law. The particles are assumed to be spherical and have an identical radius r; hence, we obtain

$$\alpha = 6\pi r\eta/V, \qquad \zeta = 4\pi r\varkappa/(Vc_{\rm f}), \tag{2}$$

where V is the characteristic volume of a particle.

The nonuniformity of the temperature field in the expressions for mass forces is taken into account on the basis of the Boussinesq approximation, i.e., the deviations of the density of the medium from the mean value are assumed to be so small that they can be neglected in all equations, except for the equation of motion, where this deviation is taken into account in the term with the lift force only. The fluid density contained in this term is expanded into a Taylor series with only two first terms of the series left:

$$\rho_{\rm f} = \rho_{f0}(1 + \beta_{\rm f}\theta). \tag{3}$$

Here $\beta_{\rm f}$ is the coefficient of thermal expansion of the fluid and $\theta = T - T_0$ is the characteristic difference in temperatures in the layer.

For consecutive allowance for the contributions of all terms in problem (1), we pass to dimensionless variables with the characteristic size h as the length scale, h^2/ν as the time scale, ν/h as the velocity scale, θ as the temperature scale, and $\rho_{f0}\nu^2/h^2$ as the pressure scale (ν is the kinematic viscosity). Then, with allowance for Eqs. (2), (3) and with changes in density of solid particles due to thermal expansion being neglected, we obtain the following equations:

$$\frac{\partial\mu}{\partial t} + \nabla\cdot(\mu\boldsymbol{v}_{\rm f}) = 0, \qquad \frac{\partial\varphi}{\partial t} + \nabla\cdot(\varphi\boldsymbol{v}_{\rm s}) = 0,$$

$$\mu\Big(\frac{\partial \boldsymbol{v}_{\rm f}}{\partial t}+\boldsymbol{v}_{\rm f}\cdot\nabla\boldsymbol{v}_{\rm f}\Big)=-\mu\nabla P+\nabla\cdot(\mu_*\boldsymbol{e})+\varphi G(\boldsymbol{v}_{\rm s}-\boldsymbol{v}_{\rm f})-\mu\,{\rm Ga}(1-\beta\theta T)(\boldsymbol{\gamma}+\boldsymbol{n}A\sin(\Omega t)),$$

$$\varphi D\Big(\frac{\partial \boldsymbol{v}_{\rm s}}{\partial t}+\boldsymbol{v}_{\rm s}\cdot\nabla\boldsymbol{v}_{\rm s}\Big)=-\varphi\nabla P-\varphi G(\boldsymbol{v}_{\rm s}-\boldsymbol{v}_{\rm f})-\varphi D\,{\rm Ga}(\boldsymbol{\gamma}+\boldsymbol{n}A\sin(\Omega t)), \tag{4}$$

$$\mu\Big(\frac{\partial T_{\rm f}}{\partial t}+\boldsymbol{v}_{\rm f}\cdot\nabla T_{\rm f}\Big)=\frac{1}{\rm Pr}\,\nabla\cdot(\mu\nabla T_{\rm f})+\varphi\zeta(T_{\rm s}-T_{\rm f}),$$

$$\varphi DB\Big(\frac{\partial T_{\rm s}}{\partial t}+\boldsymbol{v}_{\rm s}\cdot\nabla T_{\rm s}\Big)=-\varphi\zeta(T_{\rm s}-T_{\rm f}).$$

In addition to the parameter A, problem (4) has the following dimensionless parameters: the ratio of the characteristic "viscous" time to the time of equalization of velocity disturbances near the particles $G=\tau/\tau_v=(9/2)(h/r)^2\gg 1$, the ratio of the characteristic "viscous" time to the time of equalization of temperature disturbances $\zeta=\tau/\tau_T=(3/{\rm Pr})(h/r)^2\gg 1$, the Galileo number ${\rm Ga}=gh^3/\nu^2\gg 1$, the dimensionless modulation frequency $\Omega=\omega h^2/\nu$, the ratio of densities of the solid and fluid phases $D=\rho_{s0}/\rho_{f0}\gg 1$, the ratio of specific heats of the phases $B=c_{\rm s}/c_{\rm f}$, and the Prandtl number ${\rm Pr}=\nu/\chi$.

We assume that the characteristic times of equalization of disturbances near the particles are significantly smaller than the remaining characteristic times of the problem, i.e.,

$$\frac{G}{\Omega}=\frac{9}{2}\frac{\tau_\omega}{\tau_v}=\frac{9}{2}\frac{\nu}{\omega r^2}=\frac{9}{2}\Big(\frac{h_{\rm s}}{r}\Big)^2\gg 1,\qquad \frac{\zeta}{\Omega}=\frac{3}{\rm Pr}\frac{\tau_\omega}{\tau_T}=\frac{3}{\rm Pr}\frac{\nu}{\omega r^2}=\frac{3}{\rm Pr}\Big(\frac{h_{\rm s}}{r}\Big)^2\gg 1.$$

Hence, the particle radius is substantially smaller than the thickness of the viscous skin layer $h_{\rm s}=\sqrt{\nu/\omega}$. We also assume that the volume fraction of particles is so small that the value of φD asymptotically tends to zero as $D\to\infty$.

Following a standard procedure, we present the pressure, temperature, and velocity fields as the sums of constant mean values p_0, T_{f0}, T_{s0}, v_{f0}, and v_{s0} and small deviations from them p, $T_{\rm f}$, $T_{\rm s}$, $v_{\rm f}$, and $v_{\rm s}$. For these quantities, we use the following expansions into series with respect to a formal small parameter ε:

$$P=\varepsilon^{-1}P_{-1}+P_0+\ldots\qquad \boldsymbol{v}_{\rm f}=\boldsymbol{v}_{f0}+\varepsilon\boldsymbol{v}_{f1}+\ldots\ ,\qquad \boldsymbol{v}_{\rm s}=\boldsymbol{v}_{s0}+\varepsilon\boldsymbol{v}_{s1}+\ldots\ ,$$

$$\mu=1+\varepsilon^2\mu_2+\ldots\ ,\qquad \varphi=\varepsilon^2\varphi_2+\ldots\ ,\qquad T_{\rm f}=T_{f0}+\varepsilon T_{f1}+\ldots\ ,$$

$$T_{\rm s}=T_{s0}+\varepsilon T_{s1}+\ldots\ ,\qquad \tau=\Omega t,\qquad t=t_0,\qquad t_1=\Omega^{-1}t.$$

Substituting these expansions into Eq. (4) and collecting terms of the same order of smallness, we obtain the following relations in the principal order:

$$\nabla\cdot\boldsymbol{v}_{\rm f}=0,$$

$$0=-\nabla P_{-1}-{\rm Ga}(\boldsymbol{\gamma}+\boldsymbol{n}A\sin(\omega t)),\qquad 0=-\varphi G(\boldsymbol{v}_{\rm s}-\boldsymbol{v}_{\rm f})-\varphi D\,{\rm Ga}(\boldsymbol{\gamma}+\boldsymbol{n}A\sin(\Omega t)),$$

$$0=\varphi\zeta(T_{\rm s}-T_{\rm f}),\qquad 0=-\varphi\zeta(T_{\rm s}-T_{\rm f}).$$

It follows from here that the difference in temperatures of the fluid and solid phases can be neglected in the first approximation. The particle and fluid velocities are related as

$$\boldsymbol{v}_{\rm s}=\boldsymbol{v}_{\rm f}-S(\boldsymbol{\gamma}+\boldsymbol{n}A\sin(\Omega t)). \tag{5}$$

Here $S=D\,{\rm Ga}/G=(2/9)D\,{\rm Ga}(r/h)^2$ is a parameter that coincides with the parameter introduced in [6–8]. The present expansion, however, has no restrictions on the mass concentration of particles.

In the next order of expansion with allowance for Eq. (5), we finally obtain a closed system of equations of convection in a two-phase medium under conditions of finite-frequency vibrations:

$$\frac{\partial\boldsymbol{v}_{\rm f}}{\partial t}+\boldsymbol{v}_{\rm f}\cdot\nabla\boldsymbol{v}_{\rm f}=-\nabla P+\Delta\boldsymbol{v}_{\rm f}+({\rm Gr}T-\xi)(\boldsymbol{\gamma}+\boldsymbol{n}A\sin(\Omega t)),$$

$$\frac{\partial T_{\rm f}}{\partial t}+\boldsymbol{v}_{\rm f}\cdot\nabla T_{\rm f}=\frac{1}{\rm Pr}\,\Delta T_{\rm f},\qquad \frac{\partial\xi}{\partial t}+\boldsymbol{v}_{\rm f}\cdot\nabla\xi=S(\boldsymbol{\gamma}+\boldsymbol{n}A\sin(\Omega t))\cdot\nabla\xi, \tag{6}$$

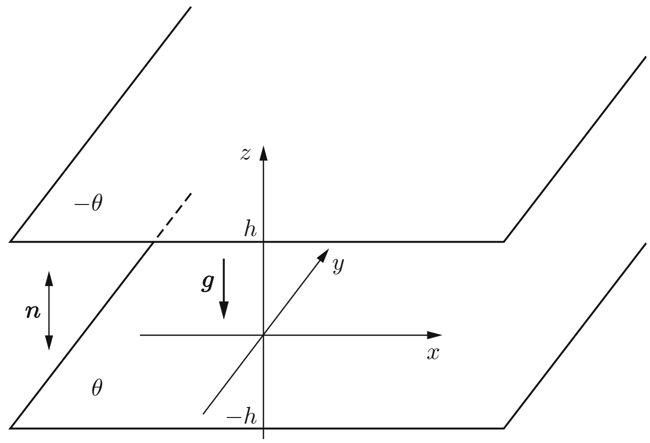

Fig. 1. Problem geometry.

$$\nabla \cdot \boldsymbol{v}_{\rm f} = 0.$$

The evolution of system (6), in addition to the previously introduced parameters S, A, Ω, and Pr, is determined by the Grashof number $\mathrm{Gr} = g\beta\theta h^3/\nu^2$. The mass concentration is $\xi = \varphi D\,\mathrm{Ga}$.

2. Stability of a Horizontal Layer of a Two-Phase Medium Under Conditions of Finite-Frequency Vibrations. We use the equations obtained above to solve the problem of stability of an infinite horizontal layer of a two-phase medium (consisting of a fluid and solid particles) of thickness $2h$ in the field of transverse finite-frequency vibrations [$\boldsymbol{n} = (0, 0, 1)$] with a constant difference in temperature at the boundaries 2θ (Fig. 1). The layer boundaries are assumed to be free (Rayleigh problem), i.e., experience no shear stresses. The boundaries are assumed to be flat, i.e., convective disturbances do not curve the boundaries. Concerning the temperature, as was noted above, its values on the boundaries are fixed; hence, there are no temperature perturbations on the boundaries. Thus, by directing the z axis vertically upward, we obtain the following system of the boundary conditions in dimensionless variables:

$$z = \pm 1\colon \qquad T_0 = \mp 1, \quad v_z = 0, \quad \frac{\partial v_x}{\partial z} = \frac{\partial v_y}{\partial z} = 0. \tag{7}$$

Problem (6) with the boundary conditions (7) admits unsteady mechanical equilibrium: the mass concentration is constant, i.e., it remains uniform over the entire volume occupied by the fluid, the velocity equals zero, and the temperature depends only on the vertical coordinate z ($T_0 = -z$). The pressure depends on both the vertical coordinate and the time:

$$P_0(z, t) = -(\mathrm{Gr} z^2/2 + \xi_0 z)(1 + A\sin(\Omega t)) + C(t).$$

Linearizing Eqs. (6) in the neighborhood of the basic state, eliminating the pressure and horizontal components of velocity, and introducing the normal disturbances of velocity $v(z, t)$, temperature $\vartheta(z, t)$, and mass concentration $\sigma(z, t)$, we obtain the following system of equations and boundary conditions:

$$\begin{gathered} \frac{\partial}{\partial t}(v'' - k^2 v) = (v^{\rm IV} - 2k^2 v'' + k^4 v) - k^2(\mathrm{Gr}\vartheta - \sigma)(1 + A\sin(\Omega t)), \\ \frac{\partial \vartheta}{\partial t} = \frac{1}{\mathrm{Pr}}(\vartheta'' - k^2\vartheta) + v, \qquad \frac{\partial \sigma}{\partial t} = S(1 + \sin(\Omega t))\sigma', \\ z = \pm 1\colon \qquad v = 0, \quad v'' = 0, \quad \vartheta = 0, \quad k^2 = k_x^2 + k_y^2. \end{gathered} \tag{8}$$

The prime here means differentiation with respect to z.

The equation for σ has the solution

$$\sigma(z, t) = \sigma_0 \sin\alpha[z + S(t - (A/\Omega)\cos(\Omega t))]. \tag{9}$$

Note that Eq. (9) is the equation for a wave propagating along the z axis with a time-dependent phase velocity, which is fairly natural: in the course of sedimentation, the particles generate a wave propagating in a medium nonuniform over its volume because of the presence of mass forces periodically dependent on time.

To further simplify the problem, we assume that the particles are so small that the parameter S characterizing particle sedimentation in the absence of vibrations can be set to zero. In this case, by virtue of symmetry, the disturbances of the mass concentration of particles on the layer boundary are expected to vanish, i.e.,

$$z = \pm 1: \qquad \sigma = 0. \tag{10}$$

With allowance for Eq. (10), problem (8) admits the exact solution

$$v = a(t)\sin(n\pi z), \qquad \vartheta = b(t)\sin(n\pi z), \qquad \sigma = \sigma_0 \sin(n\pi z), \qquad n = 1, 2, 3, \ldots\ . \tag{11}$$

Substituting Eq. (11) into Eq. (8) and assuming that $n = 1$, which corresponds to the basic level of instability, we obtain a system of ordinary differential equations for the amplitudes $a(t)$ and $b(t)$:

$$\begin{gathered}(k^2 + \pi^2)\dot{a} + (k^2 + \pi^2)^2 a = k^2(\mathrm{Gr}\, b - \sigma_0)(1 + A\sin(\Omega t)),\\ \dot{b} + (k^2 + \pi^2)b/\mathrm{Pr} = a\end{gathered} \tag{12}$$

(the dot indicates differentiation in time).

We replace the variables in Eqs. (12) in accordance with the relations

$$\frac{\partial}{\partial t} = (k^2 + \pi^2)\frac{\partial}{\partial \tau}, \quad a = (k^2 + \pi^2)\bar{a}, \quad b = \bar{b}, \quad \sigma_0 = \frac{(k^2 + \pi^2)^3}{k^2}\,\bar{\sigma}.$$

Omitting the bar over the quantities and eliminating the amplitude a, we obtain the second-order inhomogeneous equation with periodic coefficients:

$$\ddot{b} + 2\beta\dot{b} + (\omega_0^2 - A\widehat{\mathrm{Gr}}\sin(\Omega t))b = -\sigma(1 + A\sin(\Omega t)). \tag{13}$$

Here $\beta = (1 + 1/\mathrm{Pr})/2$, $\omega_0^2 = \mathrm{Pr}^{-1} - \widehat{\mathrm{Gr}}$, $\widehat{\mathrm{Gr}} = \mathrm{Gr}/\mathrm{Gr}_0$ is the reduced Grashof number, and $\mathrm{Gr}_0 = (k^2 + \pi^2)^3/k^2$ is the critical Grashof number in the absence of modulation in a pure fluid.

At $\sigma = 0$, problem (13) reduces to the problem [1] of modulation of an equilibrium vertical gradient of temperature. In this case, the solution of the system with arbitrary values of parameters either decays or increases with time. The periodic (neutral) behavior of disturbances is possible only at a certain combination of parameters, which determines the boundary of stability. Thus, finding the boundaries of convective stability reduces to finding the conditions of existence of periodic solutions of Eq. (13). At $\sigma \neq 0$, system (13) becomes inhomogeneous. Then, in addition to the parametric resonance considered in [1], the usual resonance is also possible, because the equation contains the coercive force. Moreover, at certain values of parameters, the periodic solutions found in [1] may fail to exist. To find them, certain solvability conditions have to be imposed onto the nontrivial solutions of the homogeneous problem, namely, the right side of system (13) has to be orthogonal to all solutions of the homogeneous system.

3. Expansion in the Neighborhood of the Periodic Solution. We study system (13), assuming that the decay coefficient β and modulation amplitude A are small and using the method of multiple scales. As the main idea of the method implies, we present the amplitude b, the parameters β and A, and the operator of differentiation in time in the form of series with respect to a formal small parameter ε:

$$\begin{gathered}b = b_0 + \varepsilon b_1 + \varepsilon^2 b_2 + \ldots\ , \qquad A = \varepsilon A_1 + \varepsilon^2 A_2 + \ldots\ , \qquad \beta = \varepsilon\beta_1 + \varepsilon^2\beta_2 + \ldots\ ,\\ t = t_0 + \varepsilon t_1 + \varepsilon^2 t_2 + \ldots\ , \qquad \frac{\partial}{\partial t} = \frac{\partial}{\partial t_0} + \varepsilon\frac{\partial}{\partial t_1} + \varepsilon^2\frac{\partial}{\partial t_2} + \ldots\ .\end{gathered} \tag{14}$$

Substituting Eqs. (14) into system (13) and collecting terms at identical powers of ε, we obtain an inhomogeneous problem, which has linear solutions in terms of t_0 for a certain ratio of the natural frequency and the frequency of the coercive force. Three cases are possible for the system considered: 1) $\Omega \neq \omega_0$ and $\Omega \neq 2\omega_0$; 2) $\Omega = 2\omega_0$; 3) $\Omega = \omega_0$.

In the first case, where the coercive force frequency is not multiple to the natural frequency of the system, the conditions of solvability yield exponential decay of the solution in the fundamental order of expansion. Thus, we obtain a solution oscillating with the coercive force frequency in the neighborhood of a certain mean value:

$$b = -\frac{\sigma}{\omega_0^2} - \varepsilon\,\frac{A_1\sigma}{\omega_0^2 - \Omega^2}\Big(1 + \frac{\widehat{\mathrm{Gr}}}{\omega_0^2}\Big)\sin(\Omega t_0). \tag{15}$$

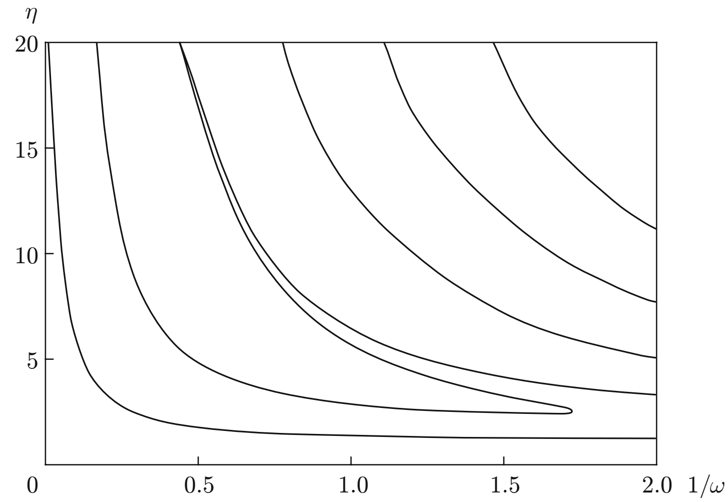

Fig. 2. Regions of stability in the plane of parameters "inverse modulation frequency — absolute modulation amplitude" at $\widehat{\mathrm{Gr}} = 1.2$ and Pr $= 1$.

In the second case, where $\Omega = 2\omega_0$, the conventional parametric resonance occurs. The solvability conditions determine the boundary separating stable solutions from unstable ones. The characteristic equation of the problem has the form

$$\lambda = -\beta_1 \pm \sqrt{\Big(\frac{\widehat{\mathrm{Gr}}A_1}{4\omega_0}\Big)^2 - \Big(\frac{\delta}{2}\Big)^2}.$$

The boundary of stability is determined from the condition $\lambda = 0$. It should be noted that the emergence of instability in the presence of dissipation, even in the case of the exact resonance with the detuning parameter $\delta = 0$, requires a finite depth of modulation of the parameter whose critical value is $A_1 = 4\beta_1\omega_0/\widehat{\mathrm{Gr}}$.

In the third case, where $\Omega = \omega_0$, the resonance can also be expected, but it has to be induced by the presence of the coercive force. Introducing the detuning parameter and assuming that $\Omega = \omega_0 + \varepsilon\delta$, we find the solution of the problem:

$$b_0 = \frac{\sigma A_1 \delta}{2\omega_0(\beta_1^2 + \delta^2)}\Big(1 + \frac{\widehat{\mathrm{Gr}}}{\omega_0^2}\Big)\cos(\Omega t_0) - \frac{\sigma A_1 \beta_1}{2\omega_0(\beta_1^2 + \delta^2)}\Big(1 + \frac{\widehat{\mathrm{Gr}}}{\omega_0^2}\Big)\sin(\Omega t_0) - \frac{\sigma}{\omega_0^2}.$$

Thus, the system tends to a steady motion with the coercive force frequency on the boundaries of stability zones determined by integer solutions.

In the case of finite values of β, we found the boundaries of stability of the basic flow of the mixture by using the Floquet method including constructing of the monodromy matrix and calculating its eigenvalues. In the calculations, we found the values of the constitutive parameters, such that at least one multiplier entered the unit circumference. The calculated results are plotted in Fig. 2 in the plane of parameters $(1/\omega, \eta)$ ($\eta = \widehat{\mathrm{Gr}}A$ is the absolute amplitude of modulation) for fixed values of the Grashof and Prandtl numbers. The value $\widehat{\mathrm{Gr}} = 1$ corresponds to the boundary of stability in the static case. In the absence of modulation, the equilibrium is unstable for $\widehat{\mathrm{Gr}} > 1$. Thus, for $\widehat{\mathrm{Gr}} = 1.2$, the axis $\eta = 0$ belongs to the region of instability. The main band of instability is adjacent to this axis; inside this band, the disturbances grow, oscillating with a frequency equal to the modulation frequency. As the modulation frequency increases, we obtain a solution oscillating in the neighborhood of a certain mean value with the coercive force frequency, which is determined by relation (15), in contrast to the case of a homogeneous fluid where stabilization is reached. A further increase in modulation amplitude leads to the emergence of regions of resonance instability with alternating integer and semi-integer solutions.

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